TWO-DIMENSIONAL SEISMIC WAVE MODELING AND INVERSION USING THE BOUNDARY ELEMENT METHOD

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INTRODUCTION: Subsurface mechanical characterization is a crucial issue in many fields of both Earth Sciences and Geotechnical Engineering. Besides the use of direct investigations, common alternative methods comprise the surface waves methods (SWM) that have been widely used for near surface characterization. This includes the original Spectral Analysis of Surface Waves (Nazarian et al. 1984; Stokoe et al. 1994), multi-channel methods (Park et al. 1999) and passive techniques (Louie, 2001). Recently, soil damping ratios were estimated as well (Rix et al. 2001). SWM assumes Rayleigh waves propagating through a stack of horizontal soil layers. Such methods are well established and computationally efficient. However they only capture the vertical variation of elastic properties. Further, the flat-layered model is only an approximation and Rayleigh waves cannot describe the scattered wave-field when strong lateral variations occur. Observations and modeling of earthquakes confirm, for example, that geometry dramatically affects the seismic wave propagation (Bard 1982/86, Raptakis et al. 2000, Savage 2004). There have been several attempts to include lateral variations by combining successive 1D inversions via the so-called "pseudo-2D" (Luo et al. 2008). However this approach still retains the 1D approximation and can only capture weak variations. SWM has been thus limited by the lack of strategies accounting more realistic soil representations. Recently, for the crustal scale such limitations have been overcome by the full waveform inversion (FWI) which exploits a finite differences (FD) forward modeling (Virieux and Operto 2009). Unfortunately, when FD is used, the subsurface is finely discretized and the number of parameters associated largely exceeds the number of measurements available leading to severely ill-posed problems that can suffer from slow convergence and instability. Here, we propose an alternative inversion formulation based on the boundary element method (BEM) that overcomes the above mentioned limitations of both FWIs and SWMs (Bignardi et al, 2011). The focus is on identifying the geometry of strongly varying interfaces in 2D layered media. The present approach is restricted to monochromatic waves of frequency ω and soil parameters are assumed known (a joint inversion is feasible but it will not be discussed here). We point out that the BEM can be easily generalized to multi-frequency. 3D geometries and viscoelasticity can be accounted for by the elastic-viscoelastic correspondence principle.

FORWARD MODEL: Consider a 2D layered system as shown in Fig. 1(a). Let Ω_j be the generic layer bounded by the curve $\Gamma = \Gamma_{j-1} \cup \Gamma_j$. The wave propagation problem in the j^{th} layer can be formulated in integral form and coupled with continuity of displacements and tensions at the layer interfaces (1).

$$\int_\Gamma \mathbf{U}^{(j)}(\mathbf{x},\mathbf{s},\omega)\mathbf{t}_{j(\mathbf{N})}(\mathbf{x},\omega)d\Gamma = \int_\Gamma \mathbf{T}^{(j)}(\mathbf{x},\mathbf{s},\omega)\mathbf{u}_j(\mathbf{x},\omega)d\Gamma + \mathbf{C}\mathbf{u}_j(\mathbf{x},\omega)\,, \quad \mathbf{x}\in\ \Gamma\,.$$

$$\mathbf{u}_j^+(\mathbf{x},\omega) = \mathbf{u}_j^-(\mathbf{x},\omega), \qquad \mathbf{t}^+_{j(\mathbf{N}^+)}(\mathbf{x},\omega) = -\mathbf{t}^-_{j(\mathbf{N}^-)}(\mathbf{x},\omega), \qquad \mathbf{x}\in\ \Gamma_j, \qquad (1)$$

where $+/-$ signs refer to the values calculated with respect to the upper/lower layer adjacent to the interface Γ_j, $\mathbf{N}$ is the outward normal. Further, vanishing normal tension $\mathbf{t}^-_{0(\mathbf{N})}$ must be enforced at the free surface Γ_0 while $\mathbf{t}^-_{0(\mathbf{N})} = f_0(\omega)\ \delta(\mathbf{x}-\mathbf{x}_0)\mathbf{z}$, is enforce at the source. f_0 is the Fourier amplitude of the load along the z axis and $\delta(\mathbf{x})$ is the Dirac function. The Green's tensors $\mathbf{U}^{(j)}$ and $\mathbf{T}^{(j)}$ and tensor $\mathbf{C}$ are given in Dominguez (1993). The BEM approximation is obtained by discretizing each interface Γ_j in e_j elements along which the field values are built by the interpolation

$$\mathbf{u}_j(\mathbf{x}) = \sum_{n=1}^{3} P_n(\eta)\mathbf{u}^{(j)}_{k,n}, \qquad \mathbf{t}_{j(\mathbf{N})}(\mathbf{x}) = \sum_{n=1}^{3} P_n(\eta)\mathbf{t}^{(j)}_{k,n}, \quad \mathbf{x}\in\ \Gamma_{j,k}\,, \quad (2)$$

where $\mathbf{u}^j_{k,n}$, $\mathbf{t}^j_{k,n}$ and $P_n(\eta)$ are displacement, traction vectors at the n^{th} node on the k^{th}. Using (2) and cycling the point of application of Green's tensors on all node positions, a system of linear equations is obtained. Equations are then assembled as in Beskos (1986). The resulting linear system is

$$[\mathbf{K}]\mathbf{u}_g = \mathbf{t}_g\,, \qquad (4)$$

where $\mathbf{u}_g$, $\mathbf{t}_g$ are global DOF. The solution of (4) follows after imposing free surface and source condition, the latter is given by $\mathbf{t}_s = m\omega^2\mathbf{d}_s + \mathbf{F}$, where the vectors $\mathbf{F}$, $\mathbf{d}_s$ and m rapresent force, displacement and mass of the source respectively.

INVERSION: Assume that the Fourier amplitudes $\mathbf{u}_{d_k}(\omega)$ of the vector displacements are known from measurements using N_r receivers and consider an equispaced discretization $\hat{\mathbf{x}}$. The shape of the interfaces can be inferred directly from the measured data by minimizing the energy functional

$$\mathcal{E}(\Gamma_j) = \frac{1}{2}\sum_{r=1}^{N_r} \left\|\mathbf{u}_k - \mathbf{u}_{d_k}\right\|^2 + \alpha \sum_{n=1}^{N_i} \sum_{k=1}^{e(n)} \int_{-1}^{1} \left[\left(\frac{\partial x_k^{(i)}}{\partial \eta}\right)^2 + \left(\frac{\partial z_k^{(i)}}{\partial \eta}\right)^2\right]^{\frac{1}{2}} d\eta \,. \quad (5)$$

The first term represents the misfit E between $\mathbf{u}_{d_k}(\omega)$ and calculated displacements $\mathbf{u}_{\mathrm{k}}(\omega)$. The second term is a regularizing term, N_i is the number of interfaces to be estimated and $e(n)$ is the number of elements of the n^{th} interface. The optimal deformation of the interfaces that makes the first variation of the energy $\mathcal{E}$ to vanish, is calculated by setting $z_j^{(i)} = z_{0,j}^{(i)} + \Delta_{z,j}$ where $z_{0,j}$ defines an initial guess and $\Delta_{z,j}$ is a small vertical perturbation. Taylor-expanding (5) to first order with respect to $\Delta_{z,j}$ yield the optimal deformation $\boldsymbol{\Delta}_z$ as a solution of the system of linear equations

$$([\mathbf{J}][\mathbf{J}]^T] + \alpha[\mathbf{R}])\{\boldsymbol{\Delta}_z\} = -[\mathbf{J}]\{\mathbf{u}_{k_0} - \mathbf{u}_{d_k}\} - \alpha\{\mathbf{q}\}, \quad (6)$$

where $\boldsymbol{\Delta}_z$ is a column vector that lists the vertical perturbations $\Delta_{z,j}$, $[\mathbf{J}]$ is the a Jacobian matrix, the superscript T denotes transpose conjugate. Matrices $[\mathbf{R}]$ and $\{\mathbf{q}\}$ are given in Bignardi (2011). Given an initial geometry, the energy (5) is minimized by iteratively evolving the interfaces according to (6) until the stopping condition is met.

APPLICATIONS: Synthetic data. Consider the system Fig. 1(b). For testing purposes data at the receivers are simulated using the forward model (4) and consist of Fourier amplitudes of both components of displacements. Gaussian noise is introduced and two set of wave velocities are considered. The first set defines a soil configuration of a high acoustic impedance jump, $V_s = 150(800)\ m/s$, $V_p = 500(2000)\ m/s$ for the upper (lower) layer, respectively, V_p being the velocity of P-waves. The second set represents a low impedance jump configuration and $V_s = 150(250)\ m/s$, $V_p = 500(1000)\ m/s$. Density $\rho = 1600(2000)\ Kg/m^3$ is fixed. Further, the source intensity is $f_0(\omega) = 10\text{^}3\ N$ at $\omega = 1\ rad/s$. The algorithm converges in about 1500 iterations, see Fig. 2(a) for both the high (curves 1,2) and low (3,4) acoustic impedance case. In Fig. 2(c) we also show the normalized misfit E/E_0 as a function of the number of iterations, E_0 being the initial misfit. In Fig. 2(a), curves (5,6) denote the converged interfaces when horizontal measurements are discarded. The algorithm can also handle irregular ground surfaces in a natural manner as illustrated in Fig. 2(b).

Field-case study. We inverted a set of experimental data collected by SWM testing at a site in Alabama in 2004. The source was an electromechanical shaker operated as a harmonic source. Vertical particle accelerations were measured by means of a linear array of 15 accelerometers located at distances ranging 2.4 to 32 meters away of the source as shown in Fig. 3(a). For different frequencies of excitation time-series were elaborated and compared to the expected displacements. SWM inversion in Fig. 3(b) and knowledge of the site lithology were exploited for the elastic parameters estimation. On purpose, we apply our algorithm in order to test if it is cabable of identifying a parallel layered structure since this model is known to be a good approximation for the site. As a result, the subsuface is revealed clearly indicating that the algorithm is capable of accurately determining the position of a flat interface as a limiting case.

CONCLUSIONS: We introduced an inversion formulation that overcomes the stiffness of classic SWMs in dealing with laterally varying subsurface layers. The proposed algorithm exploits BEM to model wave propagation through a 2D layered subsurface and infers the shape of the interfaces between the layers directly from measurements at the receivers. Tests conducted provide evidence that the geometric inversion is effective and it is applicable to geometries with steep interfaces or irregular free surfaces. Our preliminary results are very promising in reproducing irregular interfaces in a layered model. Work is in progress to extend it to multi-frequency and 3D geometries.

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